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# Ocean Radiometer for Carbon Assessment (ORCA) System Design and Radiometric Performance Analysis

Bryan Monosmith and Charles McClain

## Available Only with Approval of Issuing Office (Ocean Ecology Laboratory)

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Level of Review

This is a working paper intended to solicit comments and ideas from a technical peer group. .

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## 1 Acronym List

ACE	Aerosol/Cloud/Ecosystem	1
ACL	ACI 0301/ GIUUU/ EC03V3tCII	1

BW Band Width

CAS Computer Algebra System
CCD Charge-coupled Device
CTE Charge Transfer Efficiency
EFL Effective Focal Length

FL Focal Length

f# focal ratio or f number (FL/aperture)

IFOV Instantaneous Field OF View

LSB Least Significant Bit

NIR Near Infrared

SNR Signal to Noise Ratio

SP Super Pixel

SWIR Short Wave Infrared
TDI Time Delay Integration
QE Quantum Efficiency

#### 2 Introduction

#### 2.1 Purpose

The purpose of this technical memorandum is to describe the methods used, and the results obtained from a radiometric systems analysis of the on-orbit ORCA instrument. Beginning with an orbit, a basic optical configuration and it's transfer performance, and detector characteristics, we derive the Signal to Noise Ratio (SNR), a common and important figure of merit directly related to the quality of science data.

To allow refinements to an eventual flight instrument systems design, and to allow hardware, electronics, and other performance trades to be made in the future, it is beneficial to have a model that is also adaptable, with control over the suite of parameters that define the instrument's performance. The modeling software created, and this document, are intended to make future instrument trades and optimizations relatively easy for future instrument systems people working on ORCA.

The software used to model the on-orbit ORCA instrument was written in Microsoft Excel for desktop use (without Visual Basic macros), as well as a Computer Algebra System (CAS) called Spacetime, available for Windows, Mac, and a variety of PDA and smart phone mobile devices. Spacetime is a little known but very powerful and inexpensive package designed for rapid scripting of mathematical problems in a way that allows real-time parameter changes by way of slider bars. The spreadsheet has the advantage that all parameters that go into modeling can be seen on a large desktop monitor, allowing for a truly system view of the current design. The portable version allows for impromptu performance exploration and trades at meetings and other gatherings without the need for even a portable laptop.

#### 2.2 Instrument Background

ORCA is designed to measure phytoplankton biomass and productivity in the oceans. This is accomplished by measuring water-leaving radiances of sufficient accuracy and spectral resolution to derive data products of interest. An orbiting platform measures top-of-atmosphere radiances, of which water-leaving radiances are a small component. For the visible, Rayleigh scattering (easily modeled) and aerosol scattering account for about 90% of the measured radiance. The longer wavelength NIR and SWIR bands help correct for aerosols since open ocean absorption is very high at these wavelengths, so radiances are primarily from Rayleigh and aerosol scattering.

A requirement for the ocean color science measurement is that the instrument (specifically the detector for this analysis) not saturate at maximum radiance, which happens commonly when a highly reflective cloud is in the Instantaneous Field Of View (IFOV). This requirement allows for correction of stray light within the instrument and demands that the detector dynamic range be large enough that sufficient photons are collected at typical radiances to achieve SNR. The requirement becomes more difficult towards the red as the ratio of typical to maximum cloud radiance decreases as wavelength increases.

#### 3 Scope and Assumptions

This model study seeks to arrive at a reasonably accurate description of the expected performance of the ORCA instrument based on a conceptual baseline design and an analysis of that design. The predicted performance is based on simple, easy to understand equations and calculations. A number of assumptions have been made that are commensurate with that philosophy.

- 1. The earth has been modeled as a sphere, with higher order spherical harmonic terms equal to zero. A full treatment of a non-spherical earth will lead to small modulations of angles and distances with latitude.
- 2. The thermal environment of the instrument is benign. Effects of temperature expansion and gradients on optical beam aberration changes and misalignments are not considered.
- 3. The optical system is ideal in terms of throughput efficiency. Light that does not transmit or reflect off a surface as designed is considered as having exited the optical system. In reality, scatter and stray reflections will redirect light where it is not wanted, resulting in signal degradation.
- 4. Detector imaging is viewed as ideal. This analysis assumes all the energy from a ground element that is transmitted through the optical system is deposited entirely within the corresponding array pixel. The ORCA optical system is not diffraction limited and the f# is fast. Aberrations exist, so a fraction of the energy intended for the target pixel will invariably be deposited in other pixels. Signal photoelectrons within the array that migrate to adjacent detector pixels, for whatever reason, are not modeled.
- 5. Signal degradation due to electronics cross talk among channels is not modeled.
- 6. Cross track angles are zero. ORCA is a scanning instrument that collects data at angles up to 58° measured perpendicular to the velocity direction. All modeling in this memorandum assumes a scan angle of zero.
- 7. References to performance are "beginning of life". Instrument degradation is not considered.

Unless otherwise noted, units throughout this document are MKS.

#### 4 Definitions

#### 4.1 General Definitions

Ltyp and Lmax: These are the typical and maximum top-of-atmosphere radiances. The typical radiance is the open ocean science measurement and the maximum radiance is usually from a cloud in the IFOV. Lmax and Ltyp are functions of wavelength. Most importantly for the instrument design is that the ratio Ltyp/Lmax decreases towards the red, meaning the ocean science signal become progressively smaller at red wavelengths compared with Lmax.

Tau: The time during which a pixel accumulates signal photons. This can be either a physical pixel or a science pixel depending on the context.

Pixels: ORCA uses a detector array with a physical pixel smaller than the science pixel defined by science requirements. Data is aggregated, or binned, in both the spatial and the spectral dimensions into super pixels. The linear ratio of the science pixel size to the physical pixel size is referred to as the super pixel (SP) factor. The super pixel factors in the spatial and spectral directions need not be the same, but in ORCA they are. For the modeling documented here, "pixel" refers to the physical pixel size, NOT the aggregated science pixel size. All references to the science pixel will refer to it as such or use the term "super pixel" or SP.

TDI: Time delay Integration, described in Section 5 for the ORCA design, is the process of accumulating signal from the same ground pixel on different array pixels either during a scan (as in ORCA) or on successive scans. Subsequent addition of the pixel contents results in a larger signal than can be obtained during a single integration time. The TDI factor for ORCA is defined as (number of spatial pixels on the array)/(SP factor).

Pixel Noise: In addition to "dark" noise, "quantization" noise and "read" noise, there is the intrinsic noise arising from a data set composed of counts. There are different names for this intrinsic count noise, including "Shot", "Counting" and "Poisson" noise. They all arise when discrete events are measured by counting, and this document refers to this noise as Poisson noise.

Full Well: The capacity, in photoelectrons, of a single physical pixel. Once full well is reached, additional signal causes photoelectrons to bloom into adjacent pixel row or columns, or both.

#### 4.2 Model Parameter Definitions

The following are equation and variable definitions used in this memorandum. The definitions are idealized and are with respect to assumptions enumerated in Section 3. Orbital terms are graphically illustrated in Figure 1.

- 1. Nadir: An imaginary line from the satellite to the center of the earth. The intersection of the nadir line with the surface of the earth is the nadir point or the subsatellite point.
- 2. Orbit Altitude: the distance from the satellite to the surface of the spherical earth measured along the nadir line.
- 3. Orbital Period: The time, in minutes, a satellite takes to complete one orbit. For polar orbits, this is the time from ascending (descending) equator crossing to ascending (descending) equator crossing.
- 4. Ground Velocity: The apparent velocity of the satellite's nadir point on the surface of the earth.
- 5. Slant Angle: The angular offset of the instrument field of view centroid with respect to the nadir line, due to sensor tilt in the along track direction.
- 6. Slant Range: The distance from the satellite to the surface of the earth along the slant angle. Slant range takes curvature of the spherical earth into account.
- 7. IFOV: Instantaneous Field Of View is the angle subtended by the detector. This could be either cylindrical or rectangular, depending on the detector. For ORCA, with square pixels, the IFOV is square with the in-track IFOV equal to the cross-track IFOV.
- 8. EFL: the Effective Focal Length of the optical system.
- 9. Along Track Ground Pixel: The distance projected on the earth's curved surface, of the IFOV, in the velocity direction.
- 10. Cross Track Ground Pixel: The distance projected on the earth's surface, of the IFOV, in the cross track direction.
- 11. Ground Pixel Area: The square of the Cross Track Ground Pixel. (The line-of-sight projected area normal to the slant range line).
- 12. Aperture Solid Angle: The solid angle, in steradians, subtended by the instrument collecting area (mirror aperture) at the slant range distance.

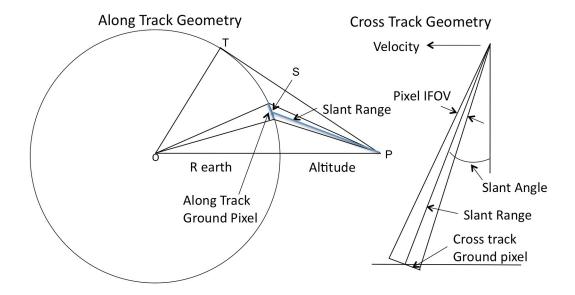


Figure 1. Orbit geometry definitions. Terms used in this memorandum. The cross track ground pixel has been rotated 90 degrees for ease of viewing.

## 5 Instrument Concept

The ORCA instrument is a scanning imaging spectroradiometer with a mirror that rotates in the cross-track direction. The off-axis mirror directs the light to a flat depolarizer and eventually focuses the light onto a slit. Since the mirror assembly rotates, the optical ray bundle rotates. In order to keep the telescope focus stationary on the slit, another mirror is needed to compensate for the rotating ray bundle. The half angle mirror accomplishes this, rotating at half the telescope rate. (Figure 2). Beyond the slit, the optical system collimates the expanding beam from the slit and redirects the light to separate blue and red channel optical modules via wavelength selective dichroic beamsplitters. The SWIR is passed to the SWIR module that uses two more dichroic beamsplitters to wavelength separate the light into 3 SWIR bands. Bandpass filters in each SWIR channel further limit the individual channel pass bands. The SWIR module is more accurately classified as a filter radiometer.

The blue and red channel modules use a reflective diffraction grating to spectrally disperse the light onto 2 dimensional arrays. The SIWR light passes through bandpass filters before being imaged onto linear arrays.

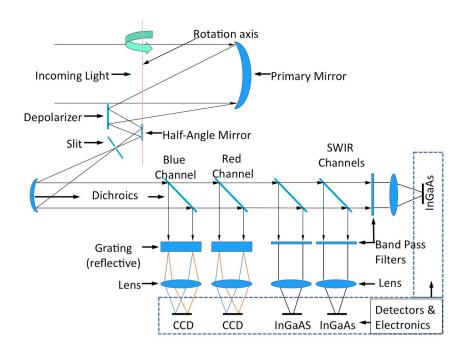


Figure 2. ORCA optical layout.

ORCA contains red and blue grating channels along with 3 SWIR bands. The primary mirror and depolarizer rotate as a single telescope unit. The half angle mirror rotates at half the telescope rate in order to keep the rotating optical ray bundle imaged onto the stationary slit. The rotation axis of the telescope assembly is contained in the plane of the page and is oriented vertically, passing through the half angle mirror face.

Upon completion of a single pixel integration time, the content of a 2-D CCD array is hyperspectral, with a strip of the ground imaged onto the array simultaneously at different wavelengths, shown schematically in Figure 3.

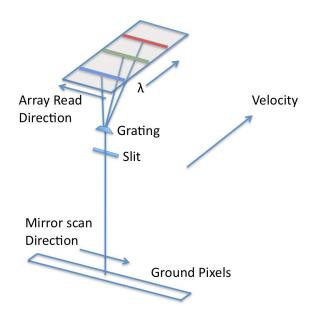


Figure 3. Ground pixel to array pixel mapping. The grating disperses the light in the along track direction with a resolution of 5nm which, when imaged onto the array, corresponds to 8 pixels.

Since the mirror rotates, the imaged ground scene will move across the array in the cross track direction. What ORCA does is match the rate of scene motion on the array due to the mirror rotation with the array column transfer rate, i.e. the rate at which photoelectrons are transferred from row to row towards the read-out register. This Time Delay Integration (TDI) technique results in an effective signal integration period equal to the single pixel integration times the number of pixels in the cross track direction on the array. TDI, along with the subsequent binning, allows for the large accumulated signal required to get signal to noise ratios of over a thousand to one.

An intermediate accumulation register sums up to 8 pixel elements before transferring the sum to the read-out register. (Figure 4) This spatial cross track binning results in a science ground pixel of 8 physical pixels in cross track length. In the velocity direction, the slit width is chosen so that the 5 nm resolution of the optical grating is mapped onto 8 pixels, which also matches the cross track science ground pixel size. This 8x8 pixel grouping, referred to as the ORCA super pixel, matches the science ground pixel as determined by top-level science requirements.

If the accumulation full well is not 8 times the single pixel well size, it is possible to accumulate 4, 2 or even 1 physical pixels so long as the readout electronics and the analog to digital converter can handle the throughput rate. In this case, the pixels are summed to the required 8 along track physical pixels in software or by a dedicated digital hardware summing circuit. Whether or not smaller accumulations with more frequent reads improves system performance is dependent on the specifics of the detector and read electronics. Larger accumulations result in loss of sensitivity given that the CCD output voltage is limited, resulting in a larger read noise. Smaller accumulations result in higher frequency operation, which increases the read noise somewhat, and also leads to more reads per superpixel.

The telescope is made to rotate with a period that matches the time it takes for the nadir point (subsatellite point) to travel a distance of 8 times the Along Track Ground Pixel. When this condition holds, there are neither gaps nor overlap in ground coverage.

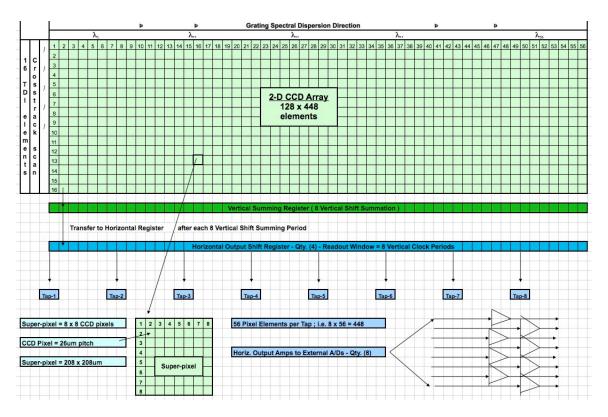


Figure 4. Visible Arrays.

The slit, corresponding to a strip of ground, is imaged (at one 5 nm spectral band) onto 128 cross track elements. The spectral information for that ground element at other wavelengths is contained in the along track direction.

The SWIR channels function by reimaging the slit onto a 16 element linear array. All the array elements are read once during each integration period. A ground spatial element moves, due to mirror rotation, a distance of one pixel on the linear array

per integration period. The SWIR bands can then take advantage of TDI by summing successive pixel elements (element 1 + element 2 + ...element 16) in software, where the added pixels are each the result of signals from the same ground pixel.

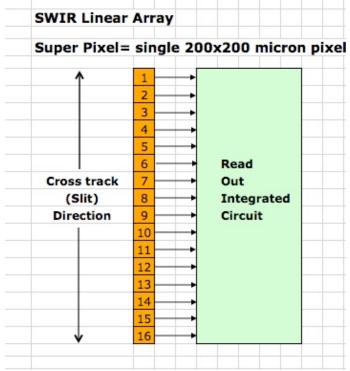


Figure 5. SWIR Arrays.

The slit is imaged across the linear array and all elements are read during an integration period. TDI accumulations are done in software.

## 6 Orbit and Earth Geometry Equations

The equations to calculate distance projections onto a sphere can be found in many books, though most are not difficult to derive. Deriving an exact formula for the Along Track Ground Pixel distance is a somewhat involved. Fortunately for the case of an orbit altitude less than 750km and a view angle less than 45 degrees, an easy approximation will suffice for ORCA's narrow IFOV. Using the approximation below, and derived in Appendix 1, results in an error in the ground super pixel of a few parts in  $10^5$ .

The Orbital Period of a polar orbiting satellite is easily derived from Kepler's third law, expressed in minutes as:

$$OrbitPeriod(minutes) = (5.2443E - 9)*(R_{earth} + OrbitAlt)^{3/2}$$

Equation 6-1

The apparent ground velocity of the nadir point is the ground circumference per period, given by:

$$GroundVelocity = \frac{2\pi * R_{earth}}{60 * OrbitPeriod(minutes)}$$
 Equation 6-2

With reference to the geometry of Figure 1, we need to know the distance from orbit to the point on the earth's surface where the line-of-sight intersects the surface. See Appendix 1 for the derivation. This distance is derived from the law of sines for triangles OTP and OSP to yield:

$$SlantRange = R_{earth} * \left[ \left( \frac{OrbitAlt}{R_{earth}} + 1 \right) * COS(SlantAngle) - \sqrt{1 - \left( \frac{OrbitAlt}{R_{earth}} + 1 \right)^2 * SIN^2(SlantAngle)} \right]$$

**Equation 6-3** 

Finally, we need to know the along track distance on the earth's curved surface seen by the IFOV of a pixel. The approximate equation, derived in Appendix 1, is:

$$AlongTrackGroundPixel = \frac{SlantRange*IFOV}{\sqrt{1 - \left(1 + \frac{OrbitAlt}{R_{earth}}\right)^{2} * \sin^{2}(SlantAngle)}}$$

**Equation 6-4** 

The Cross Track Ground Pixel is just SlantRange\* IFOV, and the view area is the square of this distance.

As discussed in Section 5, the primary mirror assembly on ORCA needs to rotate with a period that matches the time it takes to cover one Along Track Ground Pixel. For the ORCA design this is 8 times the single pixel ground distance. The mirror rotation time is:

$$Mirror Rotation Period = \left(\frac{SP*Along Track Ground Pixel}{Ground Velocity}\right)$$

**Equation 6-5** 

In summary, to compute system performance data, the geometric and orbit parameters we need are the orbital Altitude and the instrument Slant Angle.

## 7 Instrument Optics

From a systems analysis perspective, an optical system can be represented by a simple lens and detector element as in Figure 6. The detector pixel and ground pixel are in the same ratio as the EFL and altitude. The IFOV is (ground pixel)/altitude or equivalently, (detector pixel)/EFL.

Optical efficiency is necessary for calculating SNR and varies substantially with wavelength. It is measure of the fraction of light intensity that makes it to the detector face after transmission or reflection through the entire optical system.

In the context of the assumptions discussed previously, the only optical parameters of interest besides optical efficiency are the Aperture and EFL, from which we compute the aperture solid angle and the IFOV.

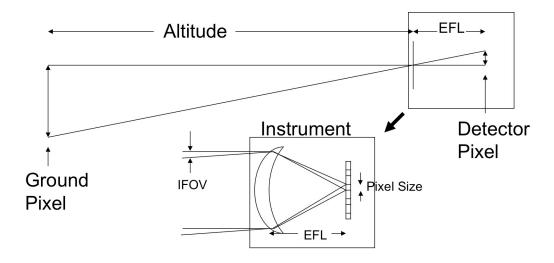


Figure 6. The simplified ORCA optics.

The optics can be represented by a lens aperture and effective focal length. The angle defined by pixel size and EFL is geometrically similar to the angle defined by altitude and ground pixel, and the value of the angle is IFOV. With reference to Figure 2, the ORCA aft optics, from the slit to the detector, are essentially reimaging optics with a magnification of 0.464. The EFL for ORCA is the primary mirror focal length multiplied by the magnification of the aft optics.

#### 8 Detector Characteristics

The important geometric information for the detector consists solely of the pixel size, assuming a square pixel. The detector pixel size and the optical EFL define the IFOV.

In addition to geometry, a calculation of SNR requires knowledge of the Read noise and the Dark current (or the equivalent if the noise is primarily Johnson noise).

Quantum Efficiency (QE) is the measure of how effectively a detector element converts power incident on the detector into photoelectrons. QE changes with wavelength and can vary from detector to detector.

Charge Transfer Efficiency (CTE) is generally included in the calculation, but in most circumstances the CTE is close enough to 1 that it can be neglected, even for 128 column transfers.

Finally, the detector well size is important, not because it enters directly into the calculation of SNR, but because it sets an upper limit for accumulated signal in cases where the system is not photon starved. For ORCA, the narrow bands in the red have SNRs limited by detector well size and bandwidth rather than signal.

#### 9 Radiometric Equations

From geometry, the power incident on an aperture from a ground area ATgP with radiance L (in units of W/sr\*m^2), at a distance SlantRange, is given by L\*( $\Omega_{aperture}$  \*Area), where  $\Omega$  is the solid angle in steradians of the aperture and Area is the projected line-of-sight area of the ground parcel. (Figure 7). By definition, there are  $2\pi$  steradians (sr) in a hemisphere. The solid angle of the aperture from a distance of SlantRange, is the ratio of the area of the aperture to the area of the half sphere of radius SlantRange, times the number of steradians in a hemisphere,

$$\Omega_{aperture} = 2\pi * \left( \frac{\pi \left( \frac{Aperture}{2} \right)^{2}}{2\pi * SlantRange^{2}} \right) = \frac{\pi}{4} * \left( \frac{Aperture}{SlantRange} \right)^{2}$$

Equation 9-1

For bandwidth limited power, when L in units of W/(sr\*micron\*m^2),

$$Power = L * \Omega_{aperture} * BW * Area_{Ground}$$

Equation 9-2

where BW is the bandwidth in the same units as L. Finally, the power actually reaching the detector surface is simply Equation 9-2 multiplied by the optical efficiency of the instrument, which results in Equation 9-3.

For a light sensitive detector, the electronic signal is proportional to the number of photoelectrons finally converted. The power incident multiplied by the time the power is applied, or the time before the photoelectrons are transferred out of the pixel, is the energy deposited. The maximum number of photoelectrons produced is the energy deposited divided by the energy per photon. Finally, in a manner similar

to the optical efficiency, the detector produces photoelectrons with an efficiency, referred to as the quantum efficiency (QE). Accounting for these factors, the final expression for the number of photoelectrons that produce the electronic signal is given by Equation 9-4.

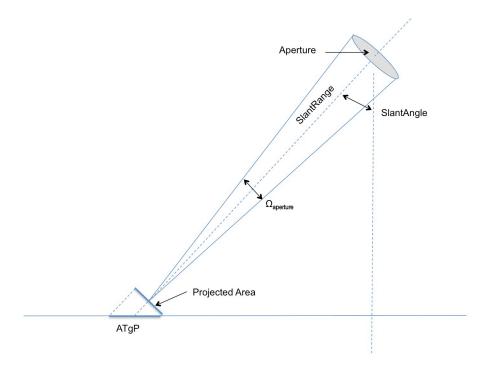


Figure 7. Geometry used to calculate detected power on the active sensor area. This simplified treatment assumes no atmosphere, whereas radiances used are top-of-atmosphere radiances. The two altitudes are inconsistent, but only by a percent or two.

$$Power_{\text{detector}} = L*BW*OptThruput*\Omega_{aperture}*Area_{Ground}$$

**Equation 9-3** 

$$PhotoElectrons = Power_{detector} * Tau * QE * \left(\frac{\lambda}{hc}\right)$$

**Equation 9-4** 

Many SWIR detector manufacturers quote photosensitivity or photoresponse in units of amps/Watt. The QE equivalent is given by:

$$QE = \frac{PhotoSensitivity*hc}{Q_{electron}*\lambda}$$

**Equation 9-5** 

#### 10 Signal to Noise Ratio

Signal noise in detector systems arises from a number of sources, including Poisson (shot) noise from both signal counts and dark counts, read noise from the electronics that convert the counts or accumulated charge into voltage, and quantization noise, due to inherent digitization limitations at the A/D stage.

Many photo detectors operate in a reverse biased voltage mode where the Poisson treatment is appropriate. Other detectors operate in a zero bias mode where the diode impedance is very low and thermal noise (Johnson noise) is an appropriate treatment. Though the dominant noise sources at the detector may have different physical mechanisms, it is convenient in this memorandum to deal with photoelectron counts by converting currents and charges into photoelectrons/sec and photoelectrons, respectively.

By convention, photoelectrons in the following equations refer to signal photoelectrons, obtained by subtracting dark counts (obtained by integrating with zero light on the detector) from the integrated total signal counts with light present.

SNR is, by definition, the signal divided by the total noise, where total noise is obtained as the square root of the sum of the squares of all noise sources, for the case where sources of noise are totally uncorrelated.

$$SNR = \frac{PhotoElectrons}{\sqrt{PhotoElectrons + Dark * Tau * SP * TDI + read^2 + quantization^2}}$$

**Equation 10-1** 

In Equation 10-1 the Poisson signal noise is given by the square root of the signal count, and the Poisson dark noise is given by the square root of the dark count, which is the Dark counts expressed as photoelectrons/s times the total integration time, which for ORCA is SP\*TDI times the single pixel integration time.

The read noise occurs once as the accumulated pixel is read by the electronics and converted to a voltage. If not defined so in the specification sheet, this can always be converted to equivalent photoelectrons.

Quantization noise is due to fundamental digitization limitations. For a single conversion the uncertainty in the A/D count is between zero and  $\frac{1}{2}$  of a least significant bit (LSB). For large signals the LSB state, and hence the error, is not correlated with the signal level and the error is uniformly distributed in this range. The variance of a uniform continuous distribution can be shown to be 1/12, over the unit range. The quantization noise is then the standard deviation of this distribution, times the count value a single LSB represents, which is:

$$QuantizationNoise = \left(\frac{FullWell}{\sqrt{12} * 2^{Bits}}\right)$$

**Equation 10-2** 

Spectral bands are constructed by aggregating pixels in the spectral direction. For ORCA, there are 8 pixels in a spectral super pixel. Bands are typically constructed from an integral number of super pixels (denoted #SPperBW), though this is not strictly required. The overall SNR is further increased, or the noise reduced, by a factor of the square root of the number of pixels aggregated spectrally. For the visible CCD detectors:

$$SNR_{Silicon} = \frac{PhotoElectrons}{\sqrt{PhotoElectrons + Dark * Tau * SP * TDI + read^2 + quantization^2}} \\ *\sqrt{\# SPperBW * SP}$$

**Equation 10-3** 

For the SWIR detectors, the physical pixel is the same size as the science pixel and spectral bandpass is constructed (or limited) by way of a filter rather than aggregating pixels in a spectrally dispersed array. The SWIR channels have TDI accumulations in software as illustrated in Figure 5. The SNR equation for the SWIR is:

$$SNR_{InGaAs} = \frac{PhotoElectrons}{\sqrt{PhotoElectrons + Dark * Tau * SP + read^2 + quantization^2}} * \sqrt{TDI}$$
 Equation 10-4

#### 11 Maximum Theoretical SNR for ORCA Visible Channels

SNR is ultimately, and obviously, about accumulating a large signal and reducing noise sources. Depending on the instrument design, one or both may be limiting factors. The initial ORCA optical design (see Section 13 ) and the TDI technique provide plenty of photons at the focal plane. This section addresses the case where the limiting factor for ORCA is the dynamic range needed in the red portion of the spectrum and the ability of the pixel to provide large enough full well capacity to meet this dynamic range requirement.

The source of the problem in the red is that the ratio, Ltyp/Lmax, decreases as wavelength increases. The requirement that the instrument not saturate at radiances of Lmax means that less of the dynamic range is available for measurement of radiances of Ltyp. To compound the problem, the important fluorescent bands are narrow spectrally and occur in the red portion of the spectrum. Narrow bands result in even less signal that can be aggregated.

This section derives a formula for the ORCA SNR as a function of bandwidth, Ltyp/Lmax ratio, full well capacity and digitization bits in order to show the limiting factors for ORCA SNR and how SNR might be improved for this design.

#### **Assumptions:**

- 1. Noise is primarily signal Poisson, with some contribution from quantization.
- 2. Dark current and read noise are both negligible. (True for baselined CCD).
- 3. Accumulation register full well capacity is SP times the single pixel capacity.

#### **Definitions:**

SignalCounts- total # of column photoelectrons after TDI and SP accumulation FullWell- single pixel well capacity r- fraction of well filled at Lmax SP- Super Pixel factor (8 for ORCA) Bits- # A/D bits (baseline is 14) L/Lmax- ratio of L to Lmax radiance BW- # of 5nm super pixel elements in a science band

Then, by definition;

$$SNR = \frac{SignalCounts}{\sqrt{SignalCounts + \left(\frac{FullWell * SP}{2^{Bits}\sqrt{12}}\right)^2}}$$

**Equation 11-1** 

Where the first term in the radical is the Poisson noise and the second term is the Quantization noise. SignalCounts can be rewritten as,

$$SignalCounts = r * \left(\frac{L}{L_{\text{max}}}\right) * FullWell * SP$$
 Equation 11-2

So if L=Lmax, and the well is at capacity (r=1), then SignalCounts=FullWell\*SP. With respect to SNR, the instrument operates in a signal limited mode when r<1, and operates in a full well limited mode when  $r \ge 1$ .

Note: r can be greater than 1, in which case the pixel overfills at Lmax and excess photoelectrons move into adjacent pixels causing signal contamination in both the spatial and spectral directions on the array.

For a single spectral column pixel, after TDI and SP accumulations;

$$SNR = \frac{r*\left(\frac{L}{L\max}\right)*FullWell*SP}{\sqrt{r*\left(\frac{L}{L\max}\right)*FullWell*SP + \frac{r*\left(\frac{L}{L\max}\right)}{r*\left(\frac{L}{L\max}\right)}*\left(\frac{FullWell*SP}{2^{Bits}*\sqrt{12}}\right)^{2}}}$$

$$SNR = \frac{r*\left(\frac{L}{L\max}\right)*FullWell*SP}{\sqrt{r*\left(\frac{L}{L\max}\right)*FullWell*SP}*\left(1 + \frac{FullWell*SP}{r*\left(\frac{L}{L\max}\right)*2^{2Bits}*12}\right)^{1/2}}$$

$$SNR = \sqrt{r * \left(\frac{L}{L \max}\right) * FullWell * SP} * \left(1 - \frac{FullWell * SP}{24r * \left(\frac{L}{L \max}\right) * 2^{2Bits}}\right)$$
Equation 11-3

For spectral band aggregated signals, SP\*BW pixel columns are summed, increasing the SNR by the square root of the quantity SP\*BW.

Finally;

$$SNR_{Band} = \sqrt{r*\left(\frac{L}{L\max}\right)*FullWell*SP^2*BW}*\left(1 - \frac{FullWell*SP}{24r*\left(\frac{L}{L\max}\right)*2^{2Bits}}\right)$$

**Equation 11-4** 

Where the first term represents the intrinsic maximum SNR for the case of an unlimited number of bits, and the second term is the degradation due to a finite number of bits.

BW need not be an integer. If one wishes to increase marginal SNR in a band, say a 10nm fluorescent band, the band can be formed by aggregating 18 instead of 16 physical pixels. BW in this case would be 2.25 (18/8).

For the ORCA red channels with typical CCD parameters (i.e. FullWell=500,000, L/Lmax=1/35), the term in parenthesis is  $\sim$ 98% for a 14 bit A/D (Bits=14). For a 12 bit A/D, the value is  $\sim$ 68%, so a 14 bit design is necessary, but little is gained with a 16 bit A/D.

The maximum SNR, without saturation, occurs when the well is at capacity for Lmax. If saturation is allowed, one can solve for r, given the other parameter values and the desired SNR.

Example: At 678nm Ltyp/Lmax=1/35.8. With 400K full well and BW=2, we have SNR=1175. Putting SNR=1500 and solving for r, we find r=1.63.

## 12 Optimizations and Trades

## 12.1 Increasing Signal

It may seem that increasing the number of photoelectrons in a detector is just a matter of increasing the aperture to gather more light. While theoretically true, increasing aperture frequently comes with unacceptable consequences. Increasing aperture without increasing the system EFL decreases the f#, which increases the magnitude of aberrations, which decreases beam quality. Attempting to control aberrations leads to aspherical surfaces or additional surfaces, both of which have disadvantages in test, complexity or ghosting.

When tasked with modifying an instrument to increase signal performance, or modifying an instrument for a different altitude, it is essential to have an understanding of the dependence of signal on various instrument and orbit parameters. The general radiometric equation can be put in a form, specific to the ORCA design, which illustrates these dependences. Begin by rewriting the radiometric equation to highlight the parameters to be traded,

$$Photoelectrons = \Gamma^* \Omega_{aperture} * Area_{Ground} * Tau$$

Equation 12-1

Where  $\Gamma$  is a term representing the product of the constant terms in Equation 9-4

From Equation 9-1, the solid angle is proportional to the square of the Aperture/SlantRange and the ground area is just the square of IFOV\*SlantRange. Tau is the angular displacement from pixel to pixel divided by the angular velocity, or IFOV/2PI\*fmirror, where fmirror is the rotational frequency, which from Equation 6-5 is GroundVelocity divided by SP\*SlantRange\*IFOV, where we have approximated the AlongTrack distance by the CrossTrack distance. The ground velocity is roughly altitude independent for the range of low earth orbits. Substituting into Equation 12-1 gives,

Photoelectrons 
$$\propto$$
 SlantRange \* Aperture<sup>2</sup> \* IFOV<sup>4</sup> or

$$Photoelectrons \propto SlantRange * Aperture^2 * \frac{PixelSize^4}{EFL^4}$$

**Equation 12-2** 

Equation 12-2 shows that additional photoelectrons for ORCA can be gotten with small increases in IFOV, which translates to small increases in the pixel size, either

the ground pixel or the array pixel, as they are equivalent. This is clear once we realize that both the area on the ground and Tau go as the square of the IFOV.

Equation 12-2 also tells us that simply decreasing the altitude of ORCA's orbit without changing the design (i.e. increasing the IFOV to keep the same ground pixel size) results in a loss of signal. What must change to avoid coverage gaps is the mirror rotation rate since this frequency goes as the inverse of (SlantRange\*IFOV).

#### 12.2 Computer Modeling

Though the Excel spreadsheet version of the performance model contains more visible information at a glance, it is cumbersome to maintain as the formulas are hidden inside cells and data references are cell based rather than variable based. The SpaceTime CAS is more portable than Excel, even running on mobile platforms such as the iPad and iPhone and Windows based PDAs. The code is written as traditional line code, with named variables, and subroutine modules, and is easily visible and maintainable. SpaceTime is better suited to explore system trades and includes features such as interactive "Scroll" bars to dynamically modify multiple instrument system parameters, with the newly calculated output values appearing immediately.

Unfortunately, SpaceTime has limited display capabilities, with output limited to a single line per module, with that having restricted commenting or annotation ability. It is designed as a computation and plotting tool, rather than a data display tool.

The following 3 screenshot captures show the various code modules, of which there are seven. Screen 1 shows module 2 where various detector characteristics are defined, including well depth, dark count rate and read noise. Module 3 in screen 2 contains all band indexed vector data such as wavelength, number of nominal 5nm bands for each science band, the actual value of the nominal 5 nm bands, typical top of atmosphere radiances, maximum radiances, optical throughput, quantum efficiency of the detector, and the baseline signal to noise ratio requirements.

Screen 2 also shows modules 4, 5 and 6, which are subroutines. The first two handle the different computations resulting from the difference between the visible spectroradiometer portion of the instrument with on-chip TDI and spectral pixel aggregations and the SWIR filter radiometer portion of the instrument with software TDI and filter defined bands. The third subroutine computes relative noise contributions from the 4 noise sources for eventual display.

Screen 3 contains module 7, which performs the bulk of the calculations outlined in Section 6 through Section 10. Screen 3 also shows the scroll bars for the following parameter inputs; TrackPixels, Altitude, visual Pixel Size, visual Effective Focal

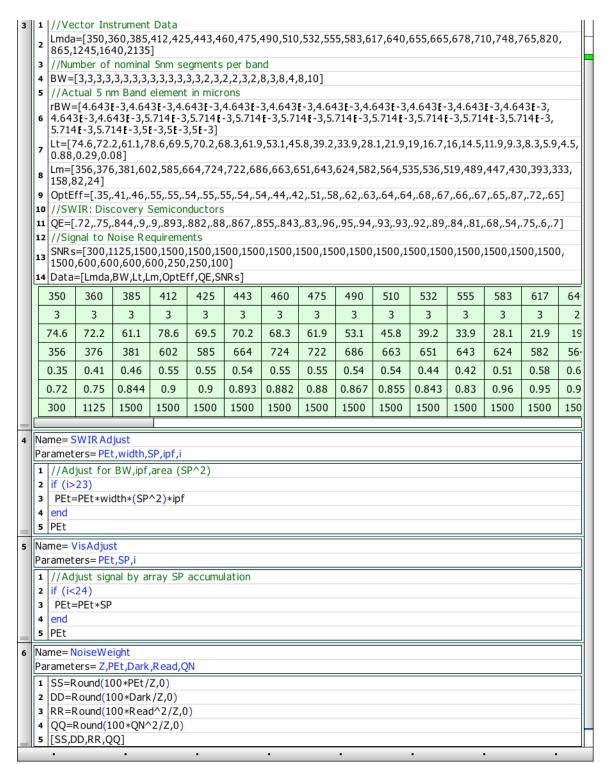
Length, instrument Aperture, Band Number, masking factor, and spectral width (in units of nominal 5nm bands).

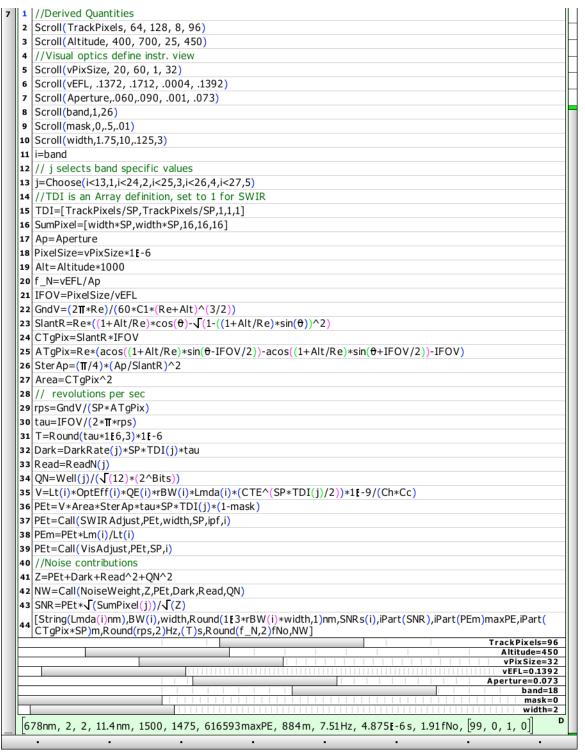
Screen 3 output, shown below the slider bars, contains the following; selected wavelength band, requirement for number of nominal 5nm bands in the science band, selected nominal width, actual bandwidth in nm, SNR requirement, and calculated SNR, photoelectrons produced at Lmax, ground science pixel size, mirror rotation rate, single visible pixel integration time, focal ratio of the visible optics, and a vector of the normalized contributions of the 4 noise sources [Signal Poisson, Dark Poisson, Read Noise, Quantization Noise].

Screen 3 shows a 450km mission with 32micron visible pixels. The selected 678nm band is set to a nominal bandwidth of 10nm (11.4nm actual). The SNR is 1475 with the constraints indicated by the slider bars and the detector parameters defined in module 2.

Screen 4 shows the 1245 nm SWIR band at 20nm bandwidth and a SNR of 413 for a 650km mission and 26 micron visible pixels. Notice in these examples, the 678 band is Poisson noise limited from signal counts whereas the 1245 band contains large noise contributions from read noise and quantization noise.

1	1	//ORCA Radiometric Performance Model, V2.4
		//Set Radian mode
		//Constants
	5	// FOV Slant angle
		<del>0</del> =20°
		// Radius of earth
		Re=6378139
		C1=5.245172 <b>[</b> -9
		Ch=6.62607 <b>F</b> -34
		Cc=Constant(light)
	$\overline{}$	99792458 <b>D</b>
2	_	//Detector Data
2		// SWIR integration period fraction
		ipf=0.8
		SP=8
		Bits=14
		CTE=.99999
		//Photoelectron well depth for [blue,red,SWIR1,SWIR2,SWIR3]
		Well=[.65[6, .65[6, 8[6, 6[6, 4[6]]
		//Dark counts/s for detectors SWIR: Discovery Semiconductors
		DarkRate=[1E3, 1E3, 1E8, 1E8, 1E8]
		//Read noise for detectors (in photoelectrons)
		ReadN=[40, 40, 200, 200, 200]
	4	0, 40, 200, 200, 200] P
//	Vec	tor Instrument Data
_		Adjust(PEt,width,SP,ipf,i)
Vis	sAd	just(PEt,SP,i) 5
No	ise\	Weight(Z,PEt,Dark,Read,QN)
//[	Deri	ved Quantities 7





```
7 1 //Derived Quantities
  2 | Scroll(TrackPixels, 64, 128, 8, 96)
  3 | Scroll(Altitude, 400, 700, 25, 450)
  4 //Visual optics define instr. view
  || 5 | Scroll(vPixSize, 20, 60, 1, 32)
  6 | Scroll(vEFL, .1372, .1712, .0004, .1392)
  | 7 | Scroll (Aperture, .060, .090, .001, .073)
  8 Scroll(band,1,26)
  9 | Scroll(mask,0,.5,.01)
  10 Scroll(width,1.75,10,.125,3)
  || 11 | i=band
  12 // j selects band specific values
  || 13 | j=Choose(i<13,1,i<24,2,i<25,3,i<26,4,i<27,5)
  14 //TDI is an Array definition, set to 1 for SWIR
  15 TDI=[TrackPixels/SP,TrackPixels/SP,1,1,1]
  || 16 | SumPixel = [width * SP, width * SP, 16, 16, 16]
  17 Ap=Aperture
  18 PixelSize=vPixSize*1[-6
  19 Alt=Altitude*1000
  20 f_N=vEFL/Ap
  21 IFOV=PixelSize/vEFL
  ||22| GndV=(2\pi*Re)/(60*C1*(Re+Alt)^(3/2))
  SlantR=Re*((1+Alt/Re)*cos(\theta)-\sqrt{(1-((1+Alt/Re)*sin(\theta))^2)}
  24 CTgPix=SlantR*IFOV
  ATgPix=Re*(acos((1+Alt/Re)*sin(\theta-IFOV/2))-acos((1+Alt/Re)*sin(\theta+IFOV/2))-IFOV)
  ||26| Ster Ap=(\Pi/4)*(Ap/SlantR)^2
  27 Area=CTgPix^2
  28 // revolutions per sec
  ||29|rps=GndV/(SP*ATgPix)
  ||30|tau=IFOV/(2*∏*rps)
  31 T=Round(tau*1[6,3)*1[-6
  32 Dark=DarkRate(j)*SP*TDI(j)*tau
  33 Read=ReadN(j)
  ||34| QN=Well(j)/(\sqrt{(12)*(2^{Bits})}
  ||35|V=Lt(i)*OptEff(i)*QE(i)*rBW(i)*Lmda(i)*(CTE^(SP*TDI(j)/2))*1[-9/(Ch*Cc)]
  36 PEt=V*Area*SterAp*tau*SP*TDI(j)*(1-mask)
  ||37|PEt=Call(SWIR Adjust,PEt,width,SP,ipf,i)
  38 PEm=PEt*Lm(i)/Lt(i)
  ||39|PEt=Call(VisAdjust,PEt,SP,i)
  40 //Noise contributions
  41 Z=PEt+Dark+Read^2+QN^2
  42 NW=Call (NoiseWeight, Z, PEt, Dark, Read, QN)
  ||43| SNR=PEt*\sqrt{(SumPixel(j))}/\sqrt{(Z)}
  String(Lmda(i)nm),BW(i),width,Round(1E3*rBW(i)*width,1)nm,SNRs(i),iPart(SNR),iPart(PEm)maxPE,iPart(
     CTgPix*SP)m,Round(rps,2)Hz,(T)s,Round(f_N,2)fNo,NW]
  TrackPixels=128
   Altitude=650
                                                                                                   vEFL=0.1392
   Aperture=0.09
   band=24
                                                                                                       width=4
   1245nm, 4, 4, 20nm, 250, 413, 5740867maxPE, 1040m, 6.08Hz, 4.887I-6s, 1.55fNo, 33, 4, 42, 21
```

## 12.3 Adapting the ORCA Design to Different Orbits

The IIP version of ORCA was developed as a 650km orbit instrument. A mission seems just as likely to be a 450km orbit mission. ORCA science advocates present SNR performance data at various venues based on the 650km IIP instrument, and though ORCA can be redesigned as a 450km instrument, or an 850km instrument, it has not been done. The purpose of this brief section is to sketch out the design changes that need to occur to transition from a 650km ORCA to a different orbit altitude. This method may not yield the optimal solution, but it at least demonstrates a solution that works with minimal impact on the current design.

The 650km ORCA has a ground pixel of 1040m, and a mirror rotation rate of 6Hz. At 450km, the same instrument will have a ground pixel of 720m and a mirror rotation rate of 9.25 Hz. The photoelectrons generated at the focal plane will be reduced by 1/3. The 650km design is not photon starved in the visible but the SWIR channels are photon starved, so this is a real problem.

As shown in Equation 12-2, a simple expression can be derived, which is ORCA specific, to relate photoelectrons produced to altitude and simple instrument parameters.

$$Photoelectrons = SlantRange * Aperture^{2} * \frac{PixelSize^{4}}{EFL^{4}}$$

In redesigning an instrument for a different application or orbit, it is preferable, if the option exists, to let two principals guide the changes.

- 1. Changes should improve performance & data quality if possible.
- 2. Changes should be low risk, i.e. easier to build, less complex, less mass/cost.

As altitude decreases, signal decreases, and if IFOV (PixelSize/EFL) doesn't change, the ground pixel gets smaller. Getting more signal by increasing Aperture is not an option because ORCA is already a fast f# system and increasing Aperture without increasing EFL decreases beam quality and violates principal 1. Increasing Aperture and increasing EFL as well (keeping f# constant) actually results in less signal. Decreasing EFL alone makes packaging optics more difficult and decreases the f#, violating principals 1 & 2.

Clearly the designer wants to increase the PixelSize. Since signal is proportional to the 4<sup>th</sup> power of PixelSize and only the square of the Aperture, one can get more signal and satisfy principals 1 and 2 by simultaneously increasing PixelSize and decreasing Aperture, while keeping the EFL constant. Reducing the Aperture makes the system f# larger, reducing aberrations and improving beam quality, especially off axis beam quality.

The simple concept is to change the primary mirror and detector imaging lenses, leaving the rest of the system unchanged, except for the new slit needed to accommodate different pixel sizes and TDI. The new primary will have an aperture and focal length both changed so as to keep the f# unchanged so that the optics beyond the slit will not see any difference in the feed beam.

Basic instrument design numbers for the 650km orbit are: EFL= 0.1392m
Aperture= 0.09m
Pixel Size=26microns

Slant ranges for the various orbits to be considered are: 450km, 481km Slant range 650km, 696km Slant range 850km, 913km Slant range

#### 450 km Orbit:

With these things in mind, we can baseline a 450km ORCA with an 884m ground pixel that is a minimal departure from the existing ORCA. The aperture will decrease to 73mm (from 90mm). The visible pixels will increase in size to 32 micron and the SWIR pixels will increase to 250 micron (from 26 and 200 respectively). These changes result in a larger field, which may stress the ability to maintain good beam quality at the field edges, though this is generally easier to maintain with larger f#s. Plugging the ratio of new to old parameters into Equation 12-2 shows the detected Photoelectrons do not change.

Modeling shows that for the 650km baseline design, the visible bands have far more signal than is required to meet SNR (generally well over 25% excess signal). The extra signal allows the optical TDI to be reduced from 16 to 12, so that the optical arrays will be 96 x 448 pixels. The diagonal edge of field will increase only slightly (with the reduced TDI) from 6.06mm to 7.33 mm, but the f# will increase from 1.55 to 1.9. The slit will be a different size to accommodate the larger pixel and new TDI for the visible (SWIR TDI will remain at 16). Calculations show the new 73mm primary will have a focal length of 243.3mm. With the original grating dispersion unchanged the focal length of the lens assemblies needs to increase by a factor of 32/26 for the CCDs (250/200 for SWIR).

#### 850km Orbit:

A similar exercise can be done for a hypothetical 850km orbit. The new pixel size can be estimated from the ratio (913km/1km)=(EFL/8\*PixelSize). The solution is 19microns. Knowing from the lower orbit example that the f# of the system is going to decrease, we let the new pixel be 22 microns. This makes the optical system a little slower at the cost of increasing the ground pixel to 1150m.

As before, use Equation 12-2 and set the photoelectrons ratio to 1. Plugging in the slant range ratios, the pixel size ratios, and solving for the aperture ratio, we get

(A850/A650)=1.22. This gives a new aperture of 110mm and a system f# of 1.26. The primary focal length is 367mm, up from the 300mm of the 650km design.

With smaller 22 micron pixels and 16 TDI, the field size at the focus is reduced, which means the image quality off-axis need not be as good as with the lower altitude missions, because the maximum off-axis angles are smaller. The optical design is nevertheless likely to be a challenge at the required 1.26 f#. Reducing the ground pixel back to 1000m makes the optics design even faster than 1.26.

As a reminder, these are simple system point designs. They are not necessarily optimal. These point designs also do not take into consideration any mechanical packaging problems introduced. The 450km orbit, in particular, has a smaller primary focal length. This means the primary must be closer to the depolarizer. If this causes an obscuration, it may necessitate a different off-axis section of the mirror curvature, increasing the angles slightly on the depolarizer and Half Angle Mirror. This would require some redesign of the front-end optics up to the collimator.

## 13 ORCA Prototype Instrument Parameters

The following are quantities for the ORCA instrument, designed for a 650km orbit (MKS units unless otherwise noted).

Altitude 650,000 Tilt angle 20° Slant Range 696,451

Orbital period 5863.70 seconds

Ground velocity 6,834.43 Aperture 0.09

Aperture steradians 1.3116e-14

Primary FL 0.3
Aft optics magnification 0.464
EFL (derived) 0.1392
Bandwidth (nominal) 5nm
Slit width 4.428e-4
Slit height 7.1724e-3
Physical pixel size 26e-6

IFOV (physical pixel) 1.86782 radian

Science pixel size 208e-6 Super pixel factor 8 TDI 16

Physical pixel ground track 130.084 (cross track)
Physical pixel ground track 140.440 (along track)
Science pixel ground track 1040.7 x 1123.5
Primary rotation rate 6.083068Hz

Integration time 4.88688e-6 (physical pixel)
Integration time 39.095 us (science pixel)

## 14 Appendix 1 - Derivations

## 14.1 Derivation of the Slant Range

From Figure 8, we use the law of sines for triangle OPT to write

$$\frac{R_{earth}}{\sin(OPT)} = \frac{R_{earth} + Alt}{1}$$

**Equation 14-1** 

and for Triangle OPS

$$\frac{R_{earth} + Alt}{\cos(e)} = \frac{R_{earth}}{\sin(SlantAngle)} = \frac{SlantRange}{\sin(L)}$$

**Equation 14-2** 

Combining Equation 14-1 and the first part of Equation 14-2,

$$\sin(SlantAngle) = \cos(e) * \sin(OPT)$$

Equation 14-3.

Noting that L+e+SlantAngle=90, Substitute angle L into the second part of Equation 14-2,

$$\frac{SlantRange}{R_{earth}} = \frac{\sin(90 - (e + SlantAngle))}{\sin(SlantAngle)} = \frac{\cos(e + SlantAngle)}{\sin(SantAngle)}$$

**Equation 14-4** 

Expanding the cos term for the sum of angles and canceling terms,

$$\frac{SlantRange}{R_{earth}} = \frac{\cos(e) * \cos(SlantAngle)}{\sin(SlantAngle)} - \sin(e)$$

**Equation 14-5** 

Rewriting the sin(e) as  $Sqrt(1-cos^2(e))$  and substituting for cos(e) from Equation 14-3,

$$\frac{SlantRange}{R_{earth}} = \frac{\cos(SlantAngle)}{\sin(OPT)} - \sqrt{1 - \left(\frac{\sin(SlantAngle)}{\sin(OPT)}\right)^2}$$

**Equation 14-6** 

Finally, substituting sin(OPT) from Equation 14-1,

$$SlantRange = R_{earth} * \left[ \left( \frac{Alt}{R_{earth}} + 1 \right) * \cos(SlantAngle) - \sqrt{1 - \left( \frac{Alt}{R_{earth}} + 1 \right)^2 * \sin^2(SlantAngle)} \right]$$

**Equation 14-7** 

## 14.2 Derivation of an Approximate Along Track Ground Pixel (ATgP)

The approximation is straightforward. From Figure 8,

$$ATgP = SlantRange * \frac{\sin(IFOV)}{\sqrt{1 - \cos^2(e)}}$$

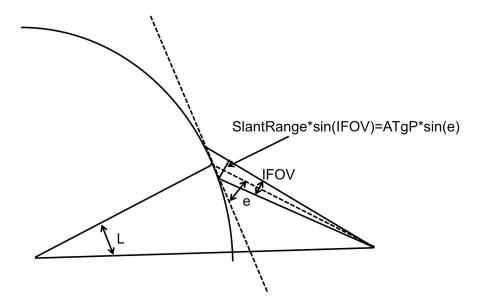
**Equation 14-8** 

where we use  $\sin^2(e) + \cos^2(e) = 1$ .

Substituting from Equation 14-3 and Equation 14-1 and noting sin(IFOV)=IFOV for ORCA's small IFOV,

$$ATgP = \frac{SlantRange * IFOV}{\sqrt{1 - \left(1 + \frac{Alt}{R_{earth}}\right)^{2} * \sin^{2}(SlantAngle)}}$$

**Equation 14-9** 



#### **Figure 8 Across Track Ground Pixel**

The approximation in the figure ignores the curvature of the earth at the ground super pixel location on the earth sphere. For orbits less than 750km & slant angles less than 45 degrees & typical ORCA IFOVs, the approximation for the Along Track Ground Pixel (ATgP) in the figure is accurate to better than a few parts in  $10^5$ .

## 14.3 Derivation of an Exact Equation for Along Track Ground Pixel

The ATgP distance is the arc along the surface of the sphere given by

$$ATgP = R_{earth} * (L_{\perp} - L_{\perp})$$

**Equation 14-10** 

where L<sub>+</sub> & L<sub>-</sub> refer to the angles formed by the intersection on the earth of the IFOV centroid+IFOV/2 and IFOV centroid-IFOV/2. Using similar definitions for e and SlantRange, again noting that L+e+SlantAngle=90, Then,

$$\frac{ATgP}{R_{earth}} = (e_{-} + SlantAngle_{-}) - (e_{+} + SlantAngle_{+}) = e_{-} - e_{+} - IFOV$$

Equation 14-11

Now use Equation 14-3 to solve for e,

$$\frac{ATgP}{R_{earth}} = a\cos\left(\frac{\sin SlantAngle_{-}}{\sin OPT}\right) - a\cos\left(\frac{\sin SlantAngle_{+}}{\sin OPT}\right) - IFOV$$

**Equation 14-12** 

Finally, substituting for sin(OPT) from Equation 14-1,

$$\begin{split} &\frac{ATgP}{R_{earth}} = a\cos\left(\left(1 + \frac{OrbitAlt}{R_e}\right) * \sin\left(SlantAngle - \frac{IFOV}{2}\right)\right) \\ &-a\cos\left(\left(1 + \frac{OrbitAlt}{R_e}\right) * \sin\left(SlantAngle + \frac{IFOV}{2}\right)\right) - IFOV \end{split}$$

**Equation 14-13** 

Calculations for various orbit altitudes, slant angles, and IFOVs confirm that the approximate ATgP differs from this exact ATgP by less than a part in  $10^5$  for any likely mission design.

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#### 13. SUPPLEMENTARY NOTES

#### 14. ABSTRACT

This technical memorandom describes the methods used and the results obtained from a radiometric systems analysis of the on-orbit ORCA instrument. Beginnning with an orbit's basic optical configurations, transfer performance, and detector characteristics; we derive the signal-to-noise ratio (SNR), a common and important figure of merit dierctly related to the quality of the science data. It is beneficial to have a model that is adaptable, allowing refinements to an eventual flight instrument systems design as well as hardware, electronics and other performance trades to be made in the future using a controllable suite of parameters that define the instrument's performance. The modeling software created and this document are intended to make future insgrument trades and optimizations relatively easy for future instrument systems operators working on ORCA.

#### 15. SUBJECT TERMS

Earth resources, remote sensing, ocean color, oceanography, biological oceanography, chlorophyll concentration, marine biology, phytoplankton concentration, instrument design, Earth sensors, sensor components, sensor characterizaton

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