Will OCI’s O$_2$ A-band channels contain the information needed to infer the height & thickness of an optically thin layer of absorbing aerosols? If not, can a MAP help?

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NO AUDIO. Please take time to read.

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Contents

• Motivation:
  ➢ UV water-leaving radiance is critical to oceanographic remote sensing.
  ➢ Atmospheric correction in the UV calls for knowledge of the height $z_{\text{top}}$ and thickness $\Delta z$ of any absorbing aerosol layer that may be present.

• Methodology
  • Forward signal model for molecular oxygen DOAS
  • Bayesian framework for estimating joint retrieval error for $\{z_{\text{top}}, \Delta z\}$

• Information Content Analysis
  • Dark surface case
  • Slightly reflective surface

• Conclusions/recommendation
OCI’s spectral sampling of Oxygen A-band
OCI’s spectral sampling of Oxygen A-band

O2 optical thickness sampling?

Nice separation

OK separation

Bad separation

N.B. “Box-car” slit function $f(\lambda)$, for simplicity.

$$\langle \tau_{O2} \rangle_{\text{band}} = \frac{\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \tau_{O2}(\lambda)f(\lambda)d\lambda}{\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} f(\lambda)d\lambda}$$

$\lambda_{\text{min}} = \lambda_{\text{start}}$

$\lambda_{\text{max}} = \lambda_{\text{min}} + 5 \text{ [nm]}$
Forward signal model: Dark surface

\[ I = \frac{\mu_0 F_0}{\pi} e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right) p_{\text{top}} / p_{\text{sfc}}} \tau_{O2,0} \]

where \( \tau_{\text{tot}} = \tau_a + \tau_{O2,0} \Delta p / p_{\text{sfc}} \)

\[ \omega_{\text{tot}} = \omega_a \tau_a / \tau_{\text{tot}} \]

\[ R(\mu_0, \mu, \varphi; b, x) = \omega_{\text{tot}} p_{\text{f}}(\mu_0, \mu, \varphi) \frac{1 - e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right) \tau_{\text{tot}}}}{4(\mu_0 + \mu)} \]

\[ p(z) \approx p_{\text{sfc}} e^{-z/8} \]

\[ \tau_{O2(z)} = \tau_{O2,0} p(z)/p_{\text{sfc}} \]
Forward signal model: Representativeness, 1

\[ I = \frac{\mu_0 F_0}{\pi} e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right)p_{\text{top}} p_{\text{sfc}}} \tau_{02,0} \]

\[ R(\mu_0, \mu, \varphi; \mathbf{b}, \mathbf{x}) \text{, with } \mu = \cos \theta \]

\[ R(\mu_0, \mu, \varphi; \mathbf{b}, \mathbf{x}) = \omega_{\text{tot}} p_{\text{f}_a}(\mu_0, \mu, \varphi) \frac{1 - e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right)\tau_{\text{tot}}}}{4(\mu_0 + \mu)} \]

where \( \tau_{\text{tot}} = \tau_a + \tau_{02,0} \Delta p / p_{\text{sfc}} \)

\[ \omega_{\text{tot}} = \omega_a \tau_a / \tau_{\text{tot}} \]

Completely black surface, which is relaxed further on.

Perfect “slab” of aerosols with uniform density. We could use another parametric model of vertical aerosol distribution such as an exponential with scale height \( H_a \) (say) \( \approx 2 \) km, or log normal profile.
Forward signal model: Representativeness, 2

$z_{TOA} = \infty$

No Rayleigh scattering above the aerosol layer, nor below either. Only molecular absorption. OK since $\tau_{Rayl} \approx 0.025$ at $O_2$ A-band wavelengths.

Only one single scattering in the aerosol layer?
OK as long as $\tau_a$ remains small, say less than $\approx 0.1$ at 765 nm, meaning $\approx 0.15$ at 550 nm.
That is precisely the scenario of interest in OCI atmospheric correction. Also aerosol SSA is < 1 in most cases of elevated layers (dust or smoke).

$p_{TOA} = 0$

$p_{top}$

$p_{top} + \Delta p$

$p_{sfc} \approx 1013.25$ mbar

$z_{top}$

$z_{top} - \Delta z$

$z_{sfc} = 0$

$I = \frac{\mu_0 F_0}{\pi} e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right) \frac{p_{top}}{p_{sfc}} \tau_{O2,0}}$

$\omega_{tot} \rho_f (\mu_0, \mu, \varphi) \frac{1 - e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right) \tau_{tot}}}{4(\mu_0 + \mu)}$

$R(\mu_0, \mu, \varphi; b, x)$, with $\mu = \cos \theta$

$R(\mu_0, \mu, \varphi; b, x) = \omega_{tot} \rho_f (\mu_0, \mu, \varphi) \frac{1 - e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right) \tau_{tot}}}{4(\mu_0 + \mu)}$

retrieved: $x = (p_{top}, \Delta p)^T$

not retr’d: $b = (\tau_a, \omega_a, \rho_f a)^T$

Cannot be separated in 1-scattering limit.
Forward signal model: Dark surface

\[ z_{TOA} = \infty \]

\[ z_{top} \approx 8 \log(p_{sfc}/p_{top}) \text{ [km]} \]

\[ \Delta z \approx 8 \Delta p/(p_{top}+\Delta p/2) \text{ [km]} \]

Differential Optical Absorption Spectroscopy (DOAS) ratio ...

\[ f(x; b) = \frac{I(\cdots; \tau_{O2})}{I(\cdots; 0)} = e^{-\left(\frac{1}{\mu_0 + \mu}\right)\tau_{O2,0} p_{top} + \Delta p / p_{sfc}} \]

retrieved: \( x = (p_{top}, \Delta p) \)

not retr’d: \( b = \tau_a \)

Just one ... pleasant surprise!

\[ p_{sfc} \approx 1013.25 \text{ mbar} \]

\[ p_{TOA} = 0 \]

\[ z_{sfc} = 0 \]

\[ z_{top} \]

\[ p_{top} \]

\[ p_{top} + \Delta p \]

\[ p_{TOA} = 0 \]

\[ p_{sfc} \approx 1013.25 \text{ mbar} \]

\[ \theta_0 \]

\[ \theta \]

\[ z_{top} \approx 8 \log(p_{sfc}/p_{top}) \text{ [km]} \]

\[ z_{TOA} = \infty \]

\[ \Delta z \approx 8 \Delta p/(p_{top}+\Delta p/2) \text{ [km]} \]

\[ D_z \approx 8 D_p/(p_{top}+\Delta p/2) \text{ [km]} \]

\[ 8 \log(p_{sfc}/p_{top}) \text{ [km]} \]

\[ p_{top} \]

\[ p_{top} + \Delta p \]

\[ p_{sfc} \approx 1013.25 \text{ mbar} \]

\[ \Delta z \approx 8 \Delta p/(p_{top}+\Delta p/2) \text{ [km]} \]
Forward signal model: Dark surface

\[ \mu_0 = \frac{1}{2}, \mu = 1; \tau_a = 0.1, p_{\text{top}} = 800, \Delta p = 200 \text{ (aerosols in PBL)} \]

\[ \tau_{O_2,0} = 0.55 \]

DOAS ratios

\[ \tau_{O_2,0} = 3.7 \]
Bayesian/Rodgers-like Estimation of Posterior/Retrieval Error & Retrievability

\[ y = \left[ \text{DOAS}_{0.5}, \text{DOAS}_{1.9}, \text{DOAS}_{2.6} \right]^T \] are data, where subscript is \( \tau_{\text{O}_2,0} \) (\( \lambda_{\text{start}} \approx 755.75 \text{ nm} \)) with \( \mu = 1 \) (\( m = 3 \)).

\[ S_y = \text{diag}((0.015y_i)^2) \] is measurement \( m \times m \) error (co)variance matrix.

\[ S_a = \text{diag}[250^2, 150^2] \] is \( n \times n \) "apriori" uncertainty (co)variance matrix in mbar\(^2\) for \( x = [p_{\text{top}}, \Delta p]^T \) (\( n = 2 \)).

\[ S_b = \left( \text{Max}[0.025, 0.15 \ tau_a] \right)^2 \] is \( 1 \times 1 \) uncertainty (variance) on non-retrieved parameter \( \tau_a \).

\( f(x, b; \cdots) \) is the forward model for the DOAS ratio.

\[ K = \frac{\partial f}{\partial x}, K_b = \frac{\partial f}{\partial b} \] are Jacobian matrices for retrieved \( (m \times 2) \) and non-retrieved \( (m \times 1) \) quantities.

\[ S_f = K_b S_b K_b^T \] is forward model error \( m \times m \) covariance matrix, from uncertainty on non-retrieved properties.

\[ S_r = S_y + S_f \] is total covariance error matrix for estimation of cost function: \( (y - f(x))^T S_r^{-1} (y - f(x))/2 \).

\[ S_x = \left( K^T S_r^{-1} K + S_a \right)^{-1} \] is the "posterior" error estimate for retrieved quantities in mbar\(^2\).

From there, uncertainty on each retrieved quantity is: \( \sigma_i = \sqrt{S_{ii}} \) (\( i = 1, \ldots, n \)) in mbar.

A non-dimensional counterpart is the "Degree of Freedom" (DoF) for each quantity: \( A_{ii} \) (\( i = 1, \ldots, n \)) from \( A = I - S_x S_a^{-1} \). Note that DoF \( \in [0, 1] \) measures what the observations do to improve on the prior info.

\[ A_{ii} > 0.7 \text{ is OK} \]

\( \approx 0.7 \text{ is OK} \)

\( \approx 0.7 \text{ is OK} \)

\( \approx 0.7 \text{ is OK} \)
Bayesian/Rodgers-like Estimation of Posterior/Retrieval Error & Retrievability

... geometric interpretation in x-space: before & after observations

Lines of equal probability density for bi-variate prior and posterior PDFs, assumed to be Gaussian, at the “1/e below max” level.

$\Delta x_2 = 4p$ [mbar]

$x_2 = \hat{x}$

$S_x$

$S_a$

$150$

$250$

$2\sigma_1$

$2\sigma_2$
DoFs of $x$, for Ocean Color Imager (OCI) only

$\mu_0 = \frac{1}{2}, \mu = 1 (\theta = 0^\circ); \tau_{O2,0} = 0.5, 1.9, 2.6; \tau_a = 0.1$

$x_1 = p_{\text{top}}$

$x_2 = \Delta p$
DoFs of $\mathbf{x}$, for OCI+MAP (Multi-Angle Polarimeter)

$$\mu_0 = \frac{1}{2}, \{\mu = 1 \ (\theta = 0^\circ)\}; \ \tau_{O2,0} = 0.5, 1.9, 2.6\}; \ {\theta = 0^\circ, 30^\circ, 60^\circ}; \ \tau_{O2,0} = 1.5}, \ \tau_a = 0.1$$

$$\begin{bmatrix} \text{DOAS}_{0.5}, \text{DOAS}_{1.9}, \text{DOAS}_{2.6} \end{bmatrix}^T$$ are data, where subscript is $\tau_{O2,0}$ with $\mu = 1 (m = 3)$.

$$\begin{bmatrix} \ldots, \text{DOAS}_0, \text{DOAS}_{30}, \text{DOAS}_{60} \end{bmatrix}^T$$ are more data, where subscript is $\theta$ with $\tau_{O2,0} = 1.5 (m = 6, \text{SPEX-like})$.

We assume a simple (MSPI2/MAIA-like) two-channel take on the A-band:

- one “in-band” channel;
- one “reference” channel near the A-band.

Note that multi-angle capability enables in principle a stereographic determination of $z_{\text{top}}$. However, for optically thin layers, smoothly distributed over horizontal directions, it may be hard to find a robust “feature” to track between views.
DoFs of $x$, for OCI+MAP (Multi-Angle Polarimeter)

$\mu_0 = \frac{1}{2}, \{\mu = 1 (\theta = 0^\circ); \tau_{O_2,0} = 0.5, 1.9, 2.6\}; \{\theta = 0^\circ, 30^\circ, 60^\circ; \tau_{O_2,0} = 1.5\}, \tau_a = 0.1$

$x_1 = p_{\text{top}}$

$x_2 = \Delta p$
Forward signal model: Partially reflective surface

From Curtis Mobley, Principal Author of:
http://www.oceanopticsbook.info/view/remote_sensing/the_atmospheric_correction_problem
Forward signal model: Partially reflective surface

\[ z_{TOA} = \infty \]
\[ z_{top} \]
\[ z_{top} - \Delta z \]
\[ z_{sfc} = 0 \]

\[ I = \frac{\mu_0 F_0}{\pi} \left[ e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right)p_{top}} \tau_{O2,0} R(\mu_0, \mu, \varphi; \tau_a, \omega_a, pf_a; x) + e^{-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right)(\tau_a + \tau_{O2,0})} \text{brf} \right] \]

retrieved: \( x = (p_{top}, \Delta p)^T \)
not retr’d: \( b = (\tau_a, \omega_a, pf_a, \text{brf})^T \)

Simple Lambertian assumption for the surface BRDF. Can be relaxed quite easily.

Only one surface reflection, and no interaction with the aerosol layer! OK since water surface BRF is small (outside of glint) and AOT is small as well.

\[ p_{TOA} = 0 \]
\[ p_{top} \]
\[ p_{top} + \Delta p \]
\[ p_{sfc} \approx 1013.25 \text{ mbar} \]
Forward signal model: Partially reflective surface

\[
\begin{align*}
\theta_0 &= \theta \\
\z_{TOA} &= \infty \\
\z_{top} &= p_{TOA} = 0 \\
\z_{top} - \Delta z &= p_{top} \\
\z_{sfc} &= 0 \\
p_{sfc} &\approx 1013.25 \text{ mbar}
\end{align*}
\]

\[
f(x; b) = \frac{I(\cdots; \tau_{O2,0})}{I(\cdots; 0)} = \frac{\omega_a \frac{1-e^{-\left(\frac{1+\mu}{\mu_0}\right)\tau_{O2,0}p_{top}+\Delta p/p_{sfc}}}{\tau_a + \tau_{O2,0} \Delta p / p_{sfc}} pf_a}{4(\mu_0 + \mu)} + \text{brf } e^{-\left(\frac{1+1}{\mu_0}\right)\tau_{sfc} \Delta p / p_{sfc}} - \left(\frac{1+1}{\mu_0}\right) \tau_{O2,0} \Delta p / p_{sfc} \right)
\]

retrieved: \( x = (p_{top}, \Delta p)^T \)

not retr’d: \( b = (\tau_a, \omega_a, pf_a, \text{brf})^T \)
Forward signal model: Dark vs non-dark surface

\[ \mu_0 = \frac{1}{2}, \mu = 1; \quad \tau_a = 0.1, \quad \omega_a = 0.9, \quad pf_a = 1, \quad p_{top} = 800, \quad \Delta p = 200; \quad \tau_{O_2,0} = 2 \]
DoFs of $x$, for OCl+MAP: Slightly reflective surface

$\mu_0 = \frac{1}{2}, \{\mu = 1 (\theta = 0^o); \tau_{O2,0} = 0.5, 1.9, 2.6\}; \{\theta = 0^o, 30^o, 60^o; \tau_{O2,0} = 1.5\}$;

$\tau_a = 0.1, \omega_a = 0.9, pf_a = 1; \text{brf} = 0.06$

Here, the max-eigenvalue of $S_f$ is ~ that of $S_y$. Need to include!

DoF plots for $p_{top}$ and $\Delta p$ should be finished for face-to-face STM. (Code is taking too long to run!) We anticipate somewhat reduced DoF values because the non-informative but uncertain surface contribution rivals the aerosol signal in magnitude. We may need more viewing angles in MAP to compensate for loss of information about aerosol layer thickness. Please ask about outcome at F2F.
Summary

• Will OCI’s O2 A-band channels contain the information needed to infer the height & thickness of an optically thin layer of absorbing aerosols?
  • Over a dark surface, aerosol layer top pressure $p_{\text{top}}$ can be retrieved with confidence, from which altitude $z_{\text{top}}$ can be derived.
  • Probably true also for a weakly reflective surface, such as turbid water.

• If not, can a MAP help?
  • Over a dark surface, aerosol layer pressure thickness $\Delta p$ will call for a MAP under most circumstances; geometric thickness $\Delta z$ can be derived from $\Delta p$ and $p_{\text{top}}$.
  • Need to extend to weakly reflective surfaces (e.g., turbid water).

• This theoretical prediction of retrieval error was based on:
  • Forward modeling with 1D linearized RT in the single-scattering/reflection limit;
  • Realistic characteristics for OCI and a notional MAP;
  • A Rodgers-like Bayesian framework for optimal estimation used to quantify the (posterior) uncertainty on retrieved properties, assuming proper convergence (no bias error) and Gaussian statistics for errors and prior, and accounting for uncertainty on non- or otherwise retrieved properties, such as AOT.